

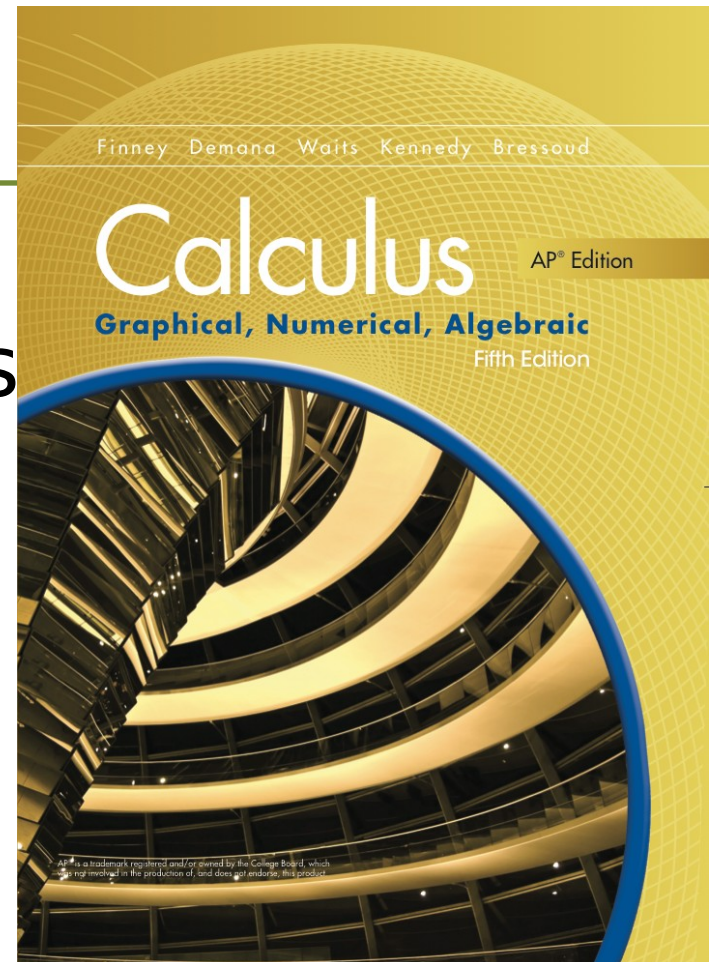
Chapter 3

Derivatives



Section 3.5

Derivatives of Trigonometric Functions



Quick Review

1. Convert 135 degrees to radians.
2. Convert 1.7 radians to degrees.
3. Find the exact value of $\sin\left(\frac{\pi}{3}\right)$ without a calculator.
4. State the domain and the range of the cosine function.
5. State the domain and the range of the tangent function.

Quick Review

6. If $\sin a = -1$, what is $\cos a$?
7. If $\tan a = -1$, what are two possible values of $\sin a$?
8. Verify the identity: $\frac{1 - \cos h}{h} = \frac{\sin^2 h}{h(1 + \cos h)}$.
9. Find an equation of the line tangent to the curve $y = 2x^3 - 7x^2 + 10$ at the point $(3, 1)$.
10. A particle moves along a line with velocity $v = 2t^3 - 7t^2 + 10$ for time $t \geq 0$. Find the acceleration of the particle at $t = 3$.

Quick Review Solutions

1. Convert 135 degrees to radians. $\frac{3\pi}{4} \approx 2.356$
2. Convert 1.7 radians to degrees. 97.403°
3. Find the exact value of $\sin\left(\frac{\pi}{3}\right)$ without a calculator. $\frac{\sqrt{3}}{2}$
4. State the domain and the range of the cosine function.
Domain: all reals Range: $[-1, 1]$
5. State the domain and the range of the tangent function.
Domain: $x \neq \frac{k\pi}{2}$ (k odd integer) Range: all reals

Quick Review Solutions

6. If $\sin a = -1$, what is $\cos a$? 0

7. If $\tan a = -1$, what are two possible values of $\sin a$? $\pm \frac{1}{\sqrt{2}}$

8. Verify the identity: $\frac{1 - \cos h}{h} = \frac{\sin^2 h}{h(1 + \cos h)}$.

Multiply by $\frac{1 + \cos h}{1 + \cos h}$ and use the identity $1 - \cos^2 h = \sin^2 h$

9. Find an equation of the line tangent to the curve

$y = 2x^3 - 7x^2 + 10$ at the point $(3, 1)$. $y = 12x - 35$

10. A particle moves along a line with velocity

$v = 2t^3 - 7t^2 + 10$ for time $t \geq 0$. Find the acceleration of the particle at $t = 3$. 12

What you'll learn about

- Derivatives of the sine and cosine functions
- Modeling harmonic motion
- Jerk as the derivative of acceleration
- Derivatives of the tangent, cotangent, secant, and cosecant functions
- Tangent and normal lines

... and why

The derivatives of sines and cosines play a key role in describing periodic change.

Derivative of the Sine Function

The derivative of the sine is the cosine.

$$\frac{d}{dx} \sin x = \cos x$$

Derivative of the Cosine Function

The derivative of the cosine is the negative of the sine.

$$\frac{d}{dx} \cos x = -\sin x$$

Example Finding the Derivative of the Sine and Cosine Functions

Find the derivative of $\frac{\sin x}{(\cos x - 2)}$.

$$\frac{dy}{dx} = \frac{(\cos x - 2) \frac{d}{dx} \sin x - \sin x \frac{d}{dx} (\cos x - 2)}{(\cos x - 2)^2} \quad \text{quotient rule}$$

$$= \frac{(\cos x - 2)(\cos x) - \sin x(-\sin x)}{(\cos x - 2)^2}$$

$$= \frac{\cos^2 x - 2\cos x + \sin^2 x}{(\cos x - 2)^2}$$

$$= \frac{(\sin^2 x + \cos^2 x) - 2\cos x}{(\cos x - 2)^2} \quad \sin^2 x + \cos^2 x = 1$$

$$= \frac{1 - 2\cos x}{(\cos x - 2)^2}$$

Simple Harmonic Motion

The motion of a weight bobbing up and down on the end of a string is an example of **simple harmonic motion**.

Example Simple Harmonic Motion

A weight hanging from a spring bobs up and down with position function $s=3\sin t$ (s in meters, t in seconds). What are its velocity and acceleration at time t ?

$$s=3\sin t$$

$$v=\frac{ds}{dt}=3\cos t$$

$$a=\frac{dv}{dt}=-3\sin t$$

Jerk

Jerk is the derivative of acceleration. If a body's position at time t is

$$j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}.$$

Derivative of the Other Basic Trigonometric Functions

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

Example Derivative of the Other Basic Trigonometric Functions

Find the equation of a line tangent to $y = x \cos x$ at $x=1$.

$$y = x \cos x$$

$$m = \frac{d}{dx}(x \cos x) = x(-\sin x) + \cos x(1)$$

Evaluate m when $x=1$

$$m = 1(-.8414709848) + (.5403023059) = -.3011686789$$

$$m = 1(-.8414709848) + (.5403023059) = -.3011686789$$

$$\text{When } x=1, y=1(\cos 1) = .5403023059$$

The equation of the tangent line is

$$y - .5403023059 = -.3011686789(x - 1)$$

$$y = -.3011686789x + .3011686789 + .5403023059$$

$$y = -.3011686789x + .8414709848$$

After rounding the equation is

$$y = -.3012x + .841$$

Example Derivative of the Other Basic Trigonometric Functions

